Introduction to : stratified flows

samples of overheads; some literature, a note

Larsen, Torben

Publication date:
2004

Document Version
Early version, also known as pre-print

5

PHD Course on
NUMERICAL AND EXPERIMENTAL ENVIRONMENTAL FLUID DYNAMICS
Aalborg University August 2004

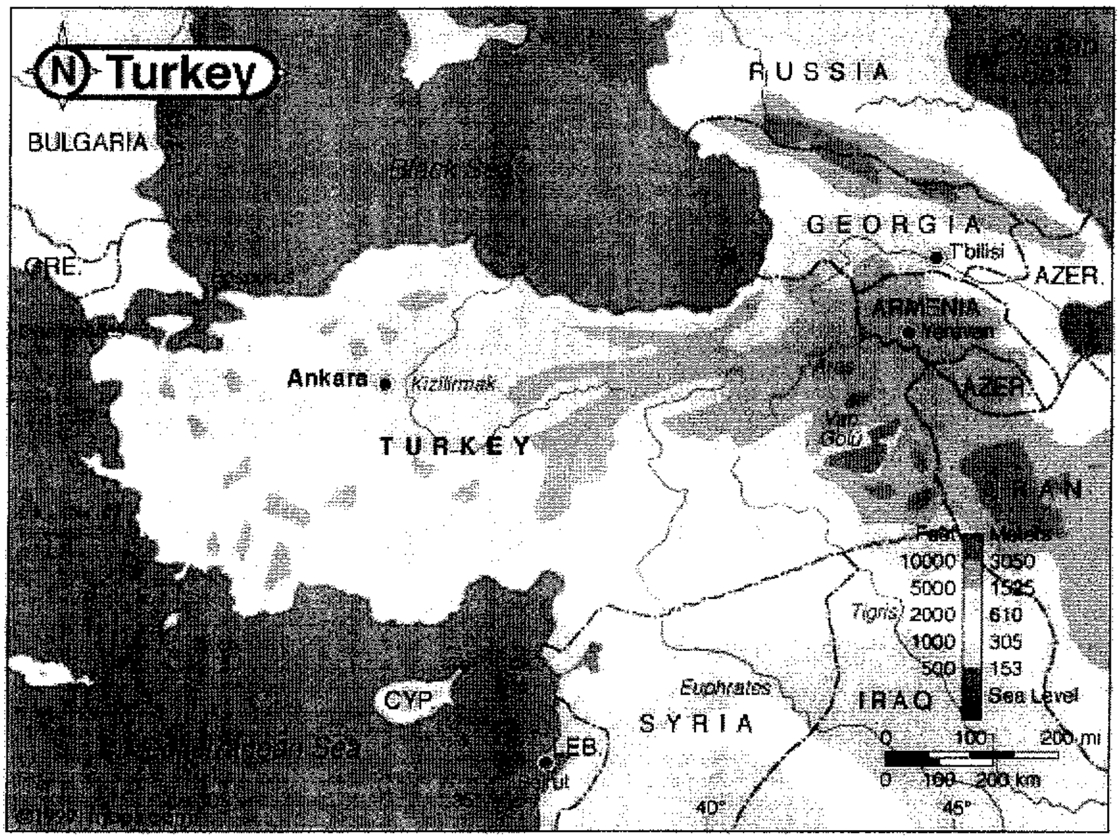
Organised by
Indoor Environmental Engineering (professor Peter V. Nielsen)
Environmental hydraulics (professor Torben Larsen)

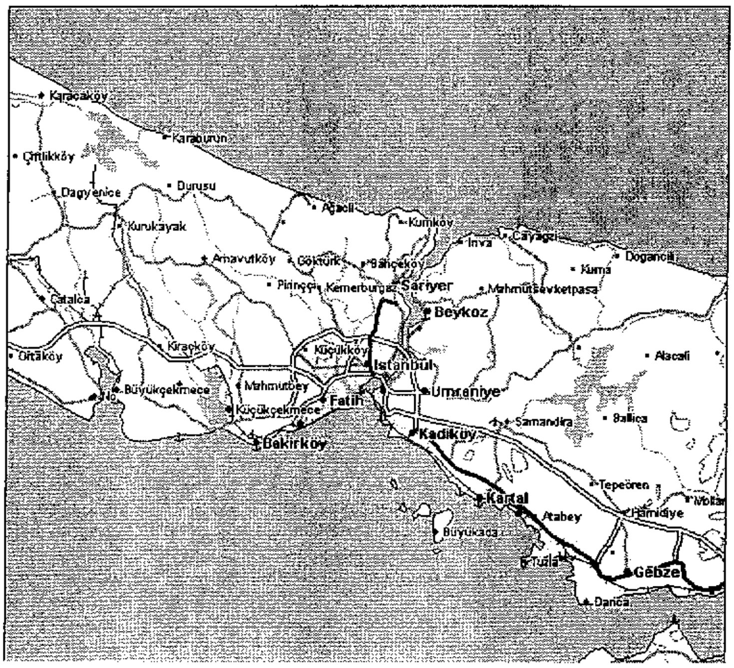
Introduction to

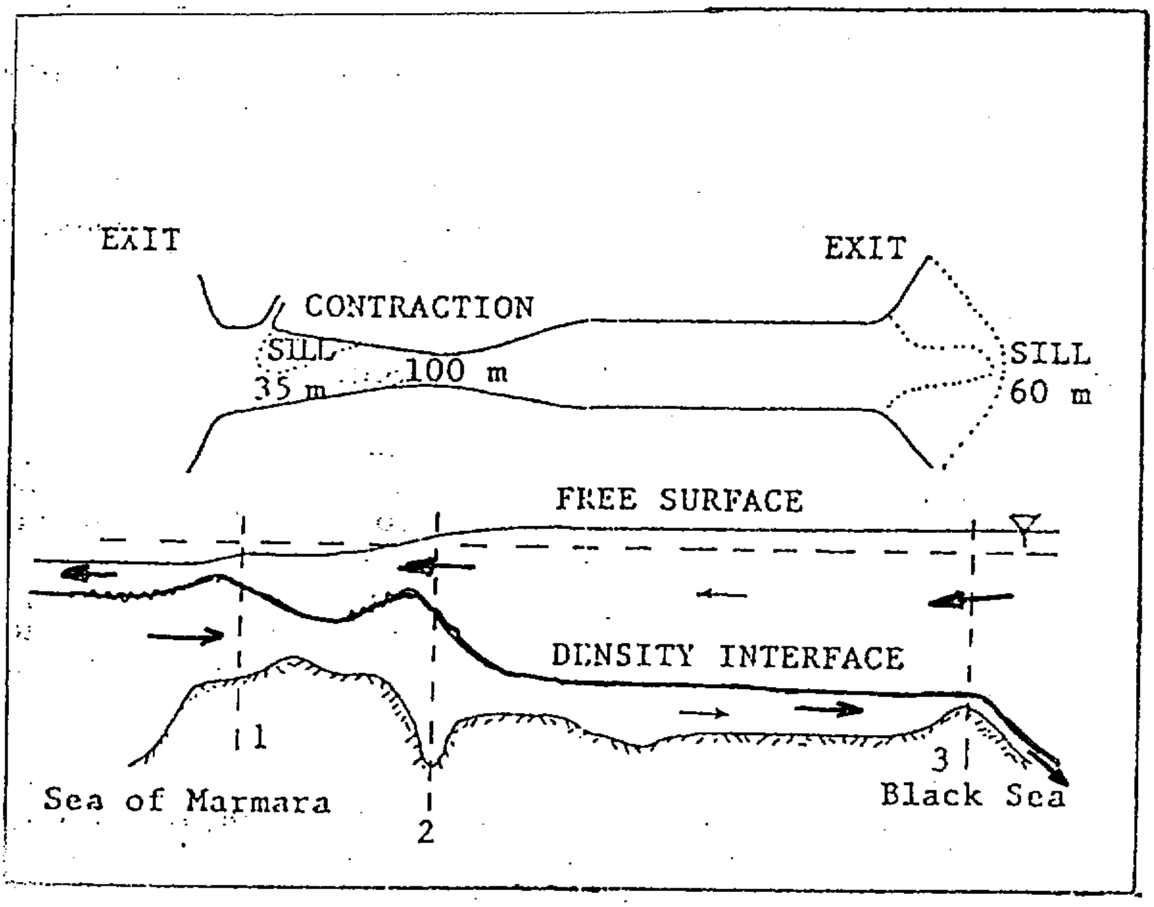
Stratified Flows

Torben Larsen

Samples of overheads
Some literature
A note


Turkey
RUSSIA
BULGARIA
GEORGIA
T'bilisi
GRE.
AZER.
ARMENIA
Yerevan
AZER.
Ankara
Kizilirmak
TURKEY
Van Gölü
IRAN
Feet
Meters
10000
3050
5000
1525
2000
610
1000
305
500
153
Sea Level
Tigris
Euphrates
IRAQ
SYRIA
CYP.
LEB.
0 100 200 mi
0 100 200 km
40°
45°


Karacaköy
Karaburun
Çiftlikköy
Dağyenice
Durusu
Ağaçli
Kumköy
Riva
Cayağzı
Doğancılı
Arnavutköy
Göktürk
Bahçeköy
Pirinççi
Kemerburgaz
Sarıyer
Mahmutşevketpaşa
Çatalca
Beykoz
Ortaköy
Kıraçköy
Küçükköy
İstanbul
Alacalı
Büyükçekmece
Mahmutbey
Ümraniye
Küçükçekmece
Fatih
Samandıra
Ballıca
Bakırköy
Kadıköy
Tepeören
Kartal
Atabey
Hamidiye
Büyükada
Tuzla
Gebze
Darıca



5. Schematic picture of the Bosphorus internal hydraulics (after Oğuz et al, 199(

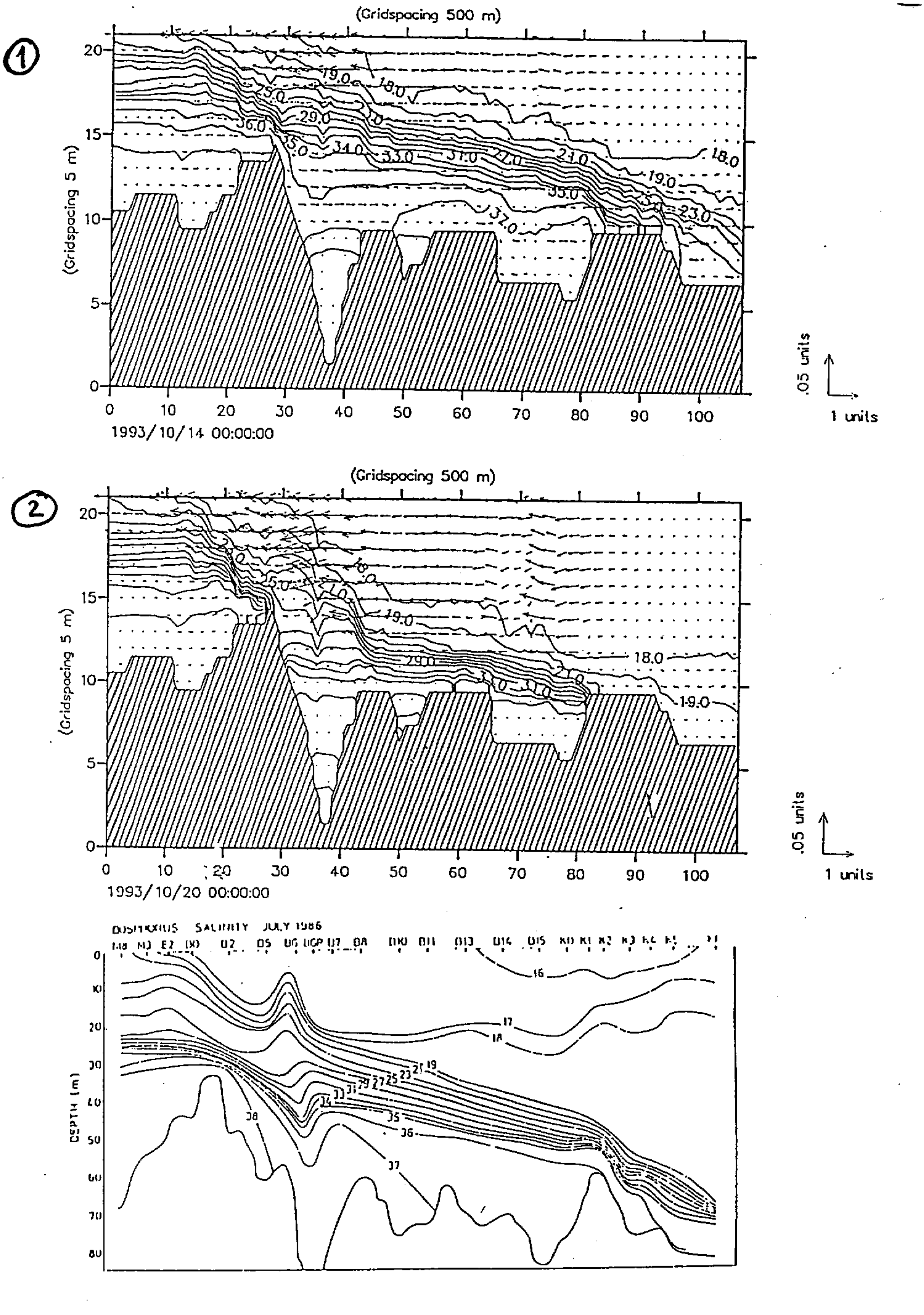
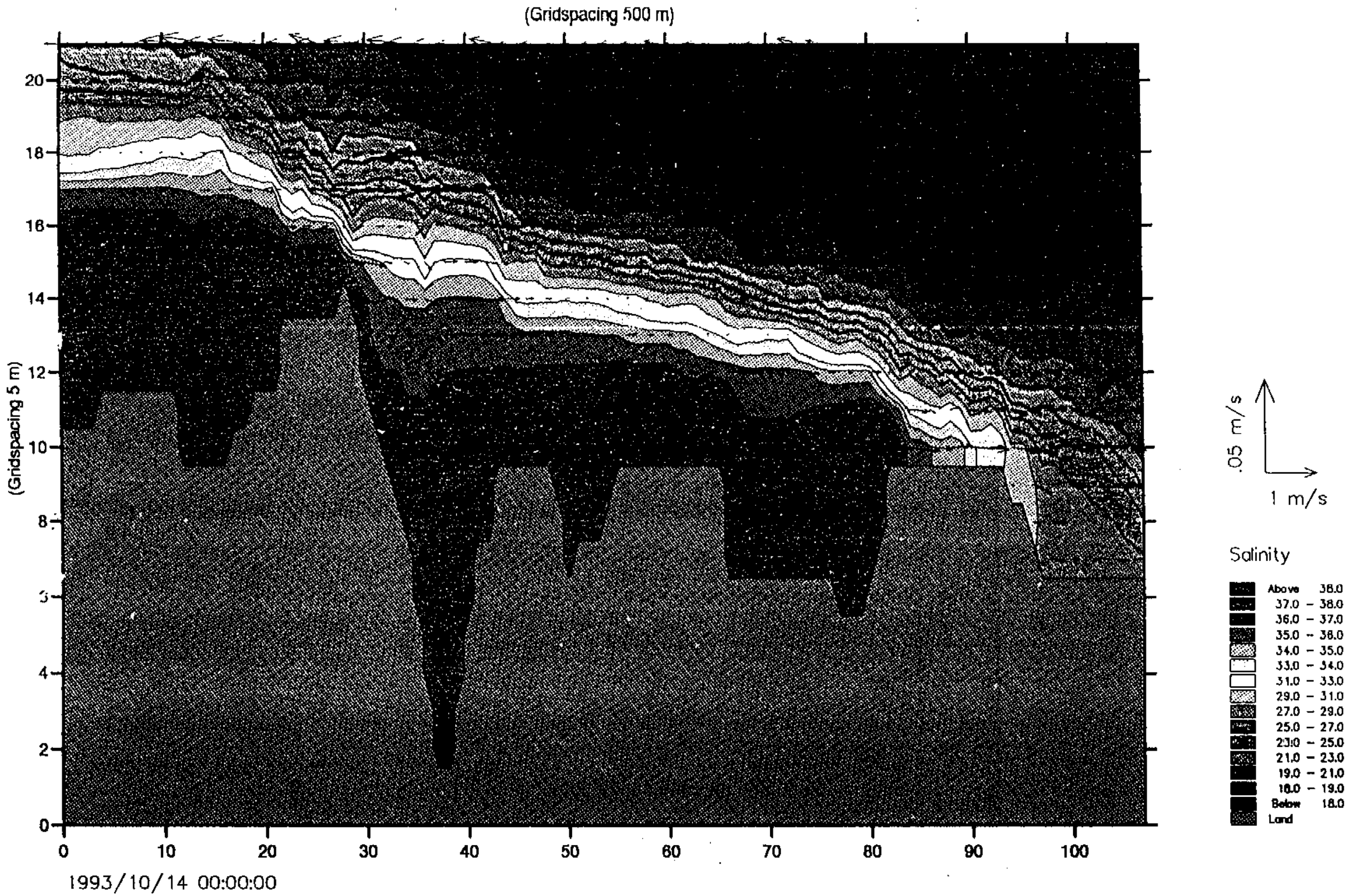


Fig 5.11 Calculated salinity transect through the Bosphorus Strait at typical flow conditions (top) and during blockage of lower layer flow (centre), and observations taken from /4/ (bottom).



Danish Hydraulic Institute		ISKI	SYSTEM3
		Istanbul Sea Modelling	
File:	Date: Mon Apr 18 1994	Bosphorus Strait Vertical cross-section October run q	dwg. no.
Scale:	Init: peb		

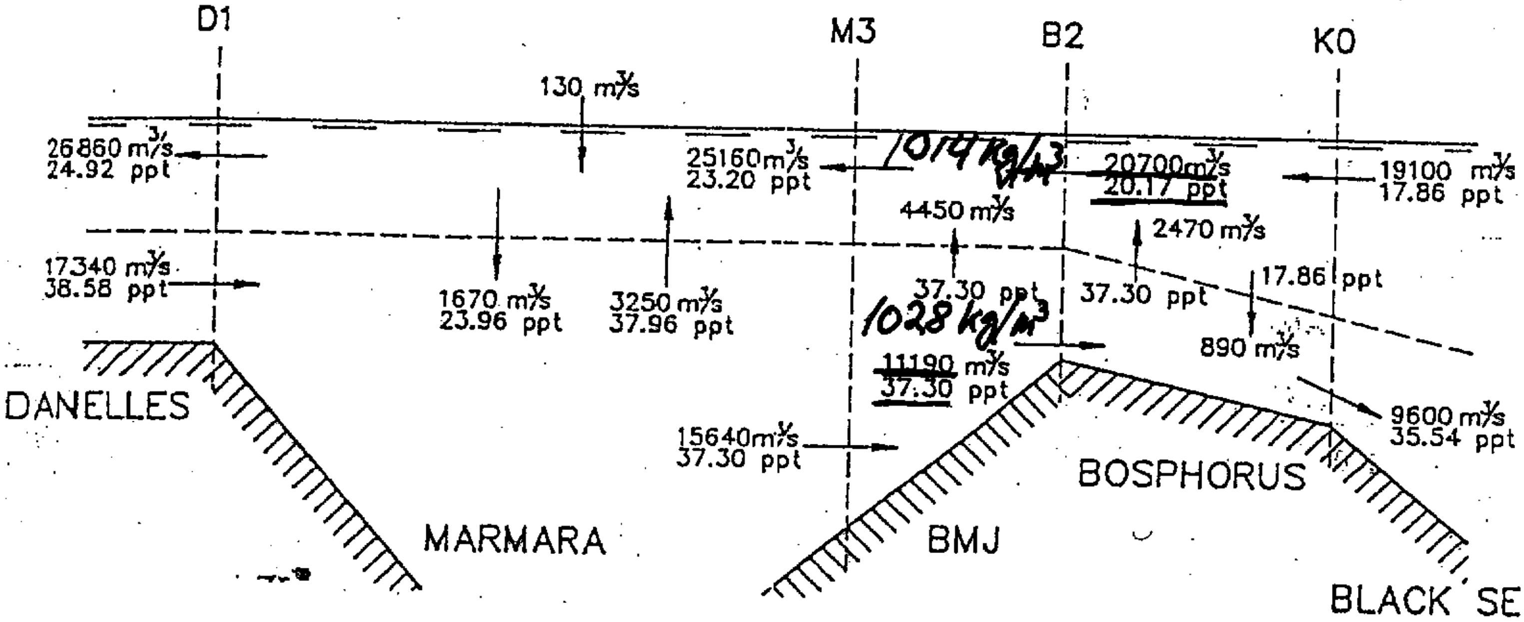
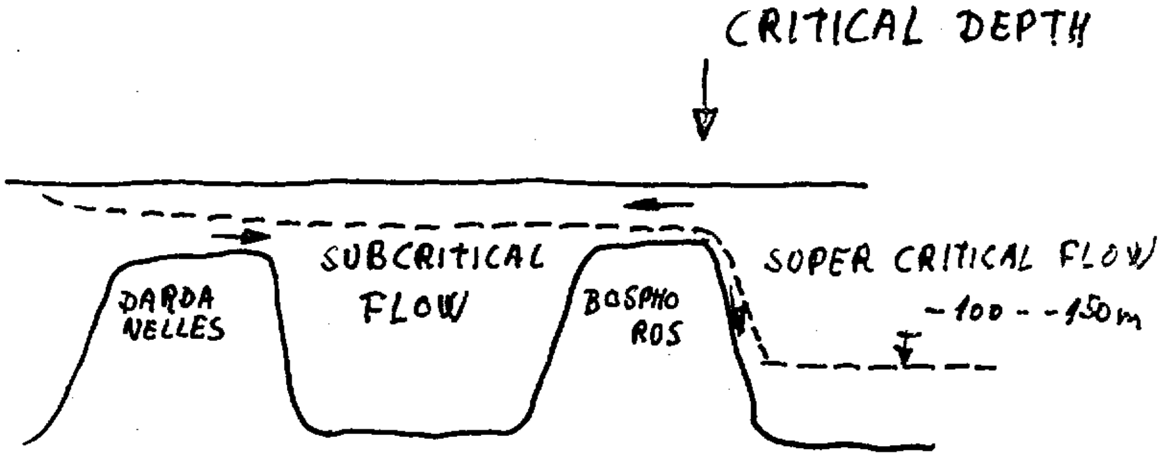
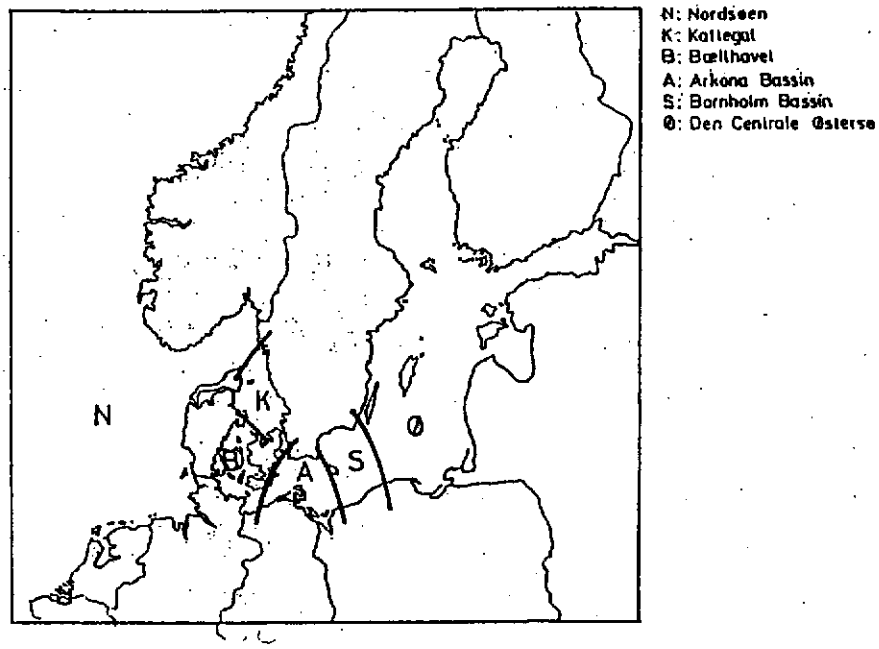


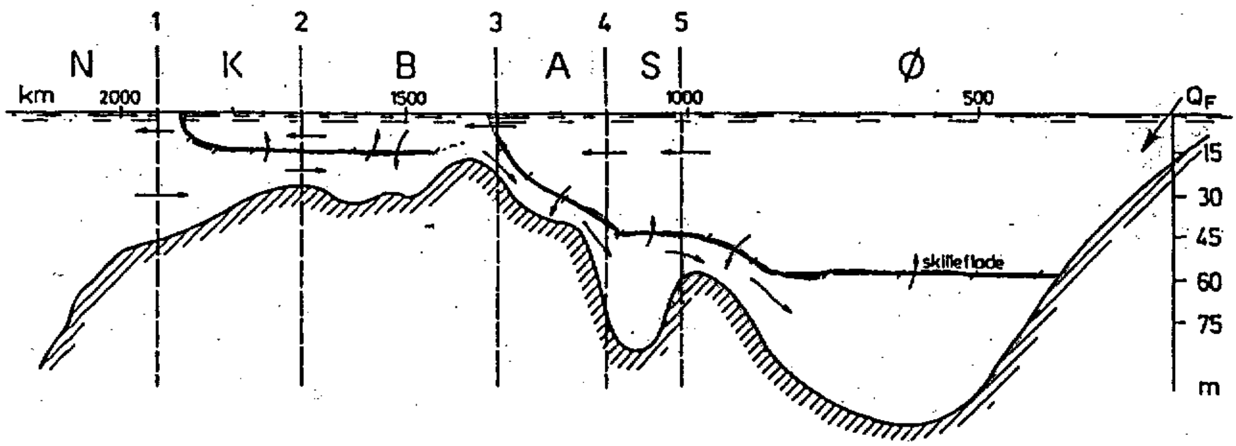
Fig. 3. Mixing along The Bosphorus and The Bosphorus/Marmara Junction (DHI, 1994)

$$\Delta\rho = \sim 14\ kg/m^3 \qquad \frac{\Delta\rho}{\rho} \approx 1,4\%$$



$$Fr_{i,d} = \frac{V}{\sqrt{\frac{\Delta \rho}{\rho} g h}} \simeq 1$$



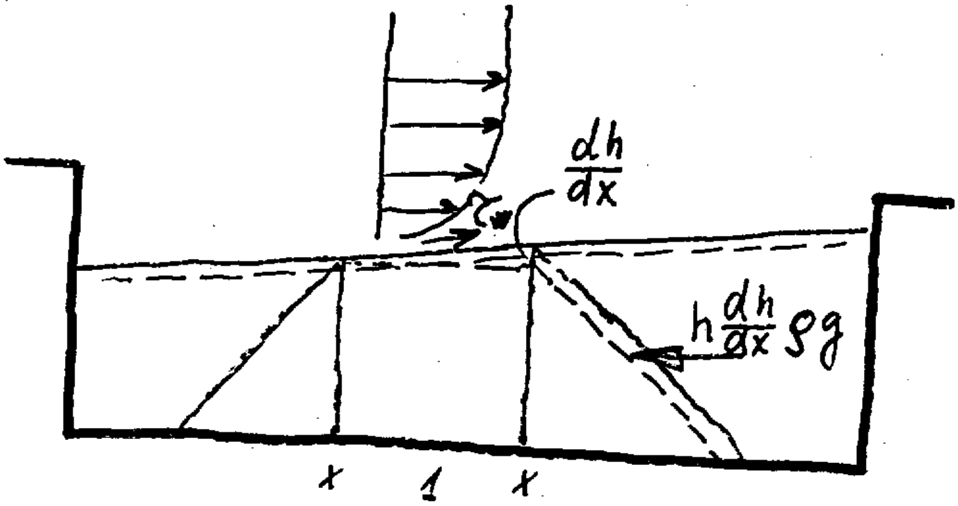


	Kattegat	2	Bælthav	3	Arkona B.	4	Bornh. B.	5	Østersø
$S_Ø$ ‰	max. 23	18	var.	8	8	8	8	7	7
S_N ‰	33	33	var.	16	var.	11	11	11	11
$Q_{eo}\uparrow$ m³/s	var.	—	$2{,}7 \cdot 10^4$	—	~0	—	$0{,}9 \cdot 10^4$	—	$2{,}6 \cdot 10^4$
$Q_{el}\downarrow$ m³/s	var	—	$2{,}3 \cdot 10^4$	—	$1{,}9 \cdot 10^4$	—	0	—	$0{,}2 \cdot 10^4$
$Q_l \leftarrow$ m³/s	var.	$3{,}3 \cdot 10^4$	var.	$2{,}9 \cdot 10^4$	var.	$4{,}8 \cdot 10^4$	var.	$3{,}9 \cdot 10^4$	var.
$Q_o \rightarrow$ m³/s	var.	$1{,}8 \cdot 10^4$	var	$1{,}4 \cdot 10^4$	var.	$3{,}3 \cdot 10^4$	var.	$2{,}4 \cdot 10^4$	var.
Q_F m³/s	~0	—	~0	—	~0	—	~0	—	$1{,}5 \cdot 10^4$

var. : varierende
max. : maximalt

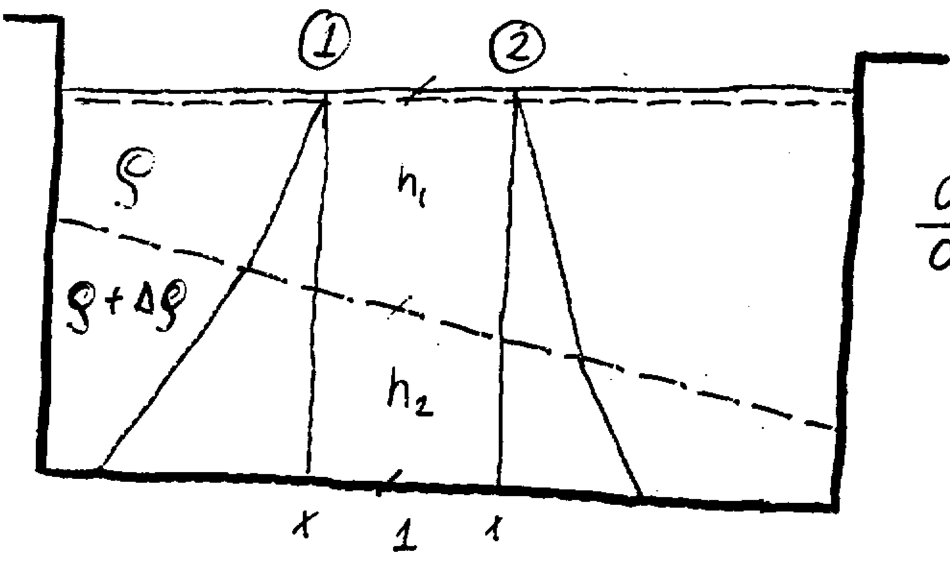
Figur 1.1.1
Snittet Leningrad - Skagerrak. Middelstrømmen og middelsaltholdighederne er angivet. Systemet er forenklet til et to-lagssystem. $S_Ø$ er saltholdigheden af øvre lag, S_N er saltholdigheden af nedre lag. Q_{eo} og Q_{el} er henholdsvis opadrettet og nedadrettet middeltransport ved blanding. Q_l og Q_o er henholdsvis middeltransporten ud af Østersøen i overfladelaget og ind i Østersøen i bundlaget. Q_F er nettotilstrømningen af ferskvand. (Bo Pedersen og Møller 1981).

WIND SET-UP



$$\tau_w = h\frac{dh}{dx}\rho g \Rightarrow \frac{dh}{dx} = \frac{\tau_w}{h\rho g}$$

WIND SET-UP IN STRATIFIED ESTUARY



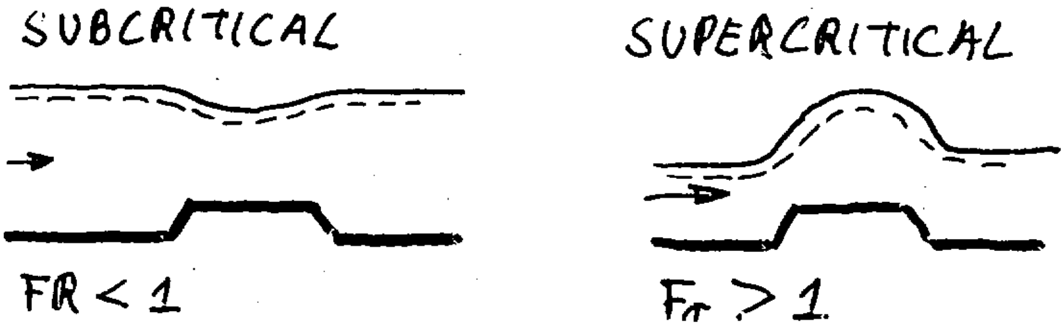
$$\frac{dh}{dx} = \frac{\tau_w}{h_1\rho g}$$

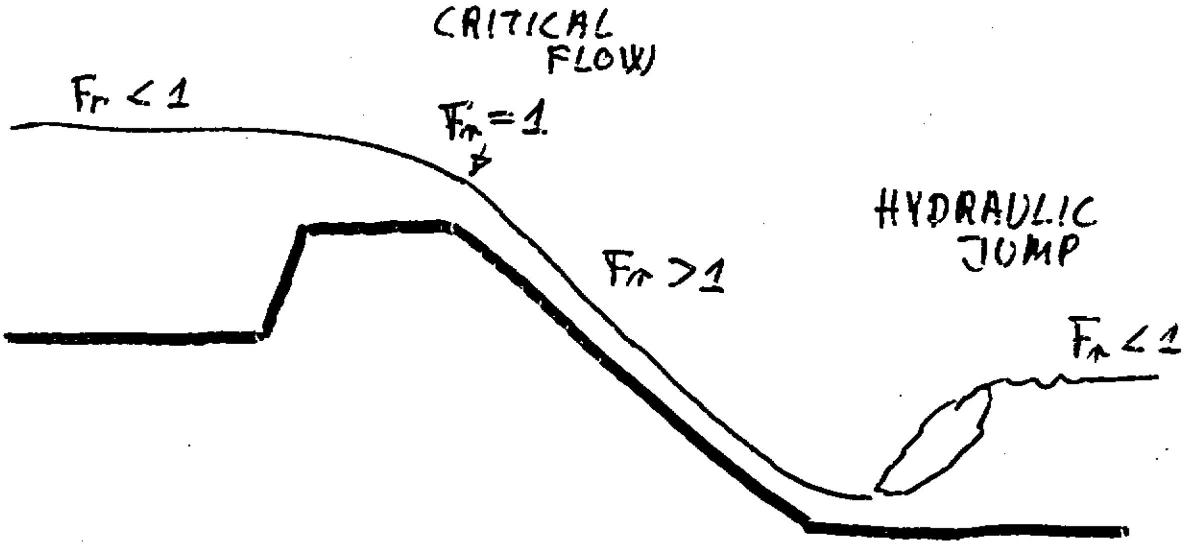
$$h_1\rho g + h_2(\rho+\Delta\rho)g = \left(h_1+\frac{\partial h_1}{dx}\right)\rho g + \left(h_2+\frac{dh_2}{dx}\right)(\rho+\Delta\rho)g$$

$$\frac{dh_2}{dx} = -\frac{dh}{dx}\frac{\rho}{\Delta\rho}$$

$$40 < \frac{\rho}{\Delta\rho} < \infty$$

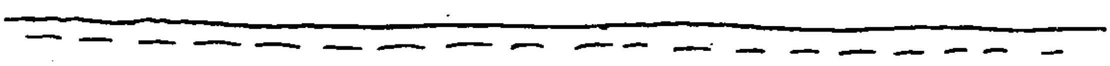
FROUDE NUMBER $Fr = \frac{u}{\sqrt{g h}}$

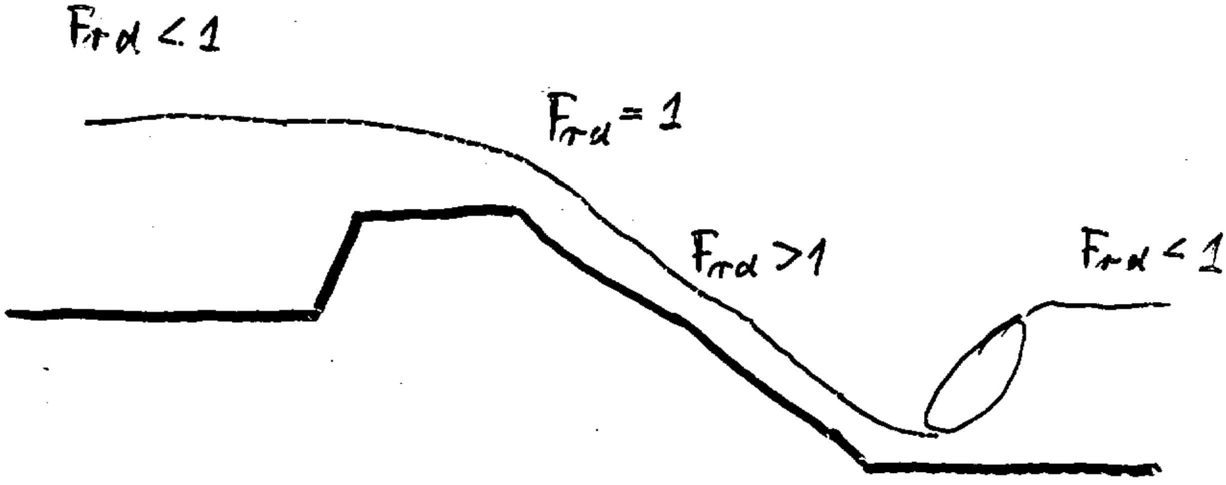




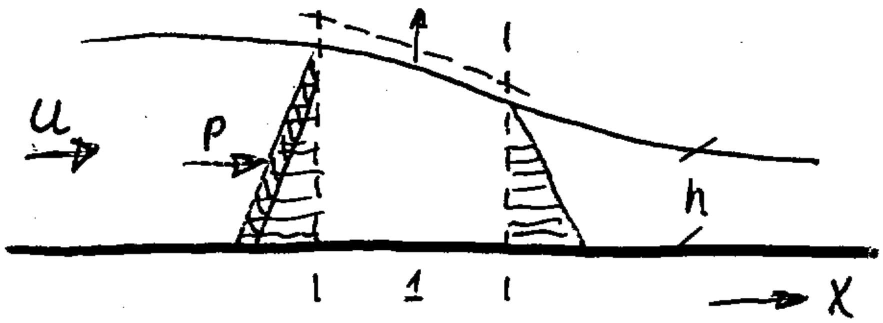
DENSIMETRIC FROUDE NUMBER $Fr_{,d} = \frac{u}{\sqrt{g \frac{\Delta\rho}{\rho} h}}$

INTERNAL FLOW





LONG SURFACE WAVE



CONTINUITY

$$h\frac{\partial u}{\partial x} + \frac{\partial h}{\partial t} = 0$$

MOMENTUM X-DIR.

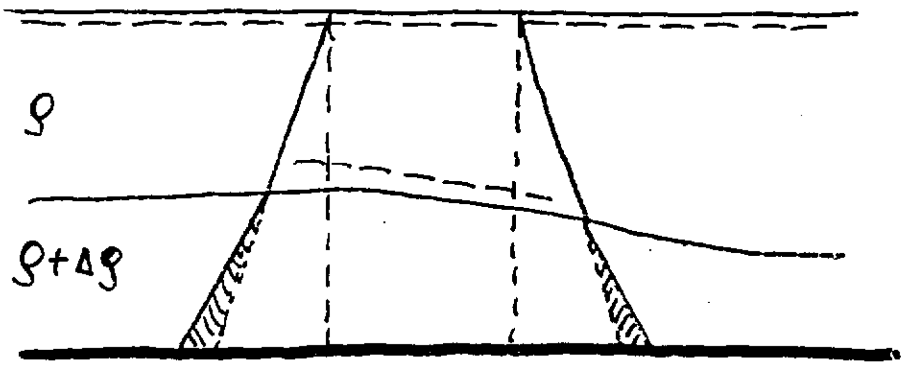
$$h\rho\frac{\partial u}{\partial t} + h\rho g\frac{\partial h}{\partial x} = 0$$

WAVE EQ.

$$\frac{\partial^2 h}{\partial t^2} = c^2\frac{\partial^2 h}{\partial x^2}$$

$$c = \pm\sqrt{gh}$$

INTERNAL WAVE



CONTINUITY

$$h\frac{\partial u}{\partial x} + \frac{\partial h}{\partial t} = 0$$

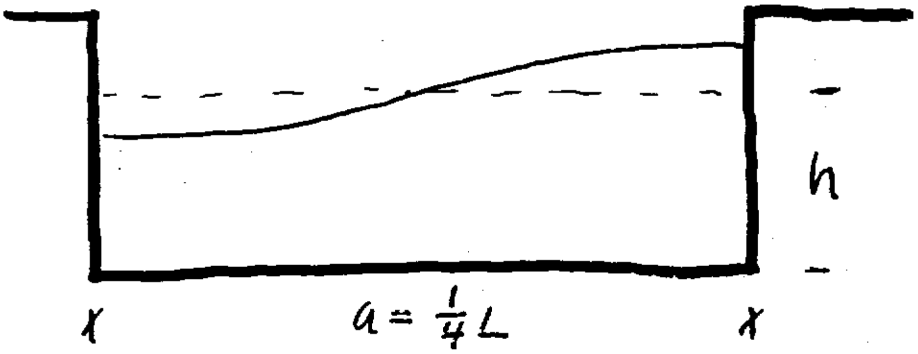
MOMENTUM

$$h\rho\frac{\partial u}{\partial t} + h\Delta\rho g\frac{\partial h}{\partial x} = 0$$

$$\frac{\partial^2 h}{\partial t^2} = c^2\frac{\partial^2 h}{\partial x^2}$$

$$c = \pm\sqrt{g\frac{\Delta\rho}{\rho}h}$$

FREE OSCILLATIONS IN CLOSED BASIN



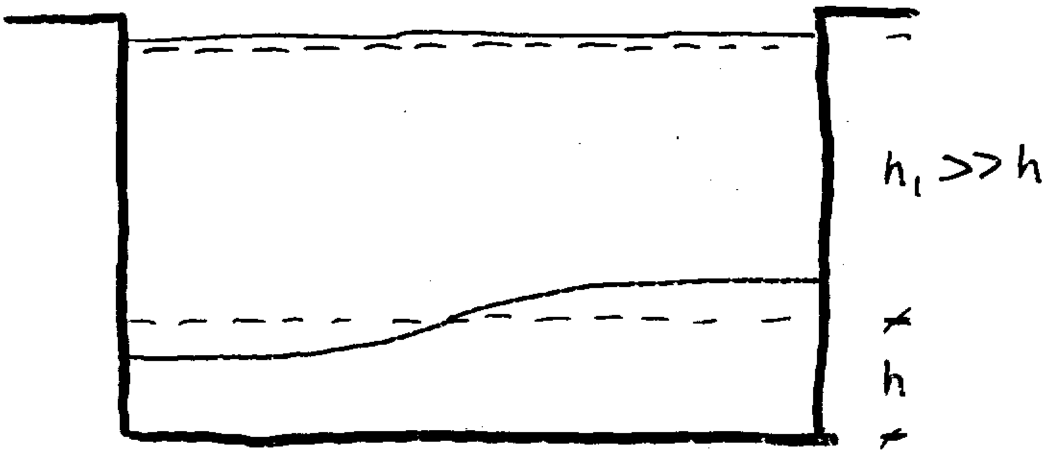
$a = \frac{1}{4} L$

$L = 4a$

$C_s = \frac{L}{T} \quad , \quad C_s = \sqrt{g\,h}$

$T_s = \frac{4a}{\sqrt{g\,h}}$

STRATIFIED BASIN



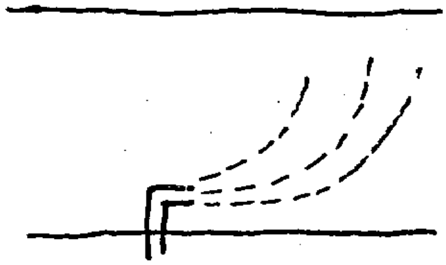
$C_i = \sqrt{g \frac{\Delta\rho}{\Delta\rho} h}$

$T_i = \frac{4a}{\sqrt{g \frac{\Delta\rho}{\rho} h}}$ $\qquad \frac{\Delta\rho}{\rho} \sim 2\% \Rightarrow T_i \simeq 7 \cdot T_s$

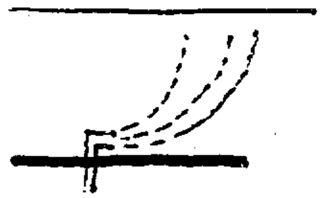
JETS AND PLUMES

SIMILARITY

PROTOTYPE



MODEL

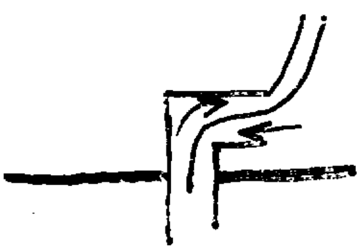


$$F_{rd} = \frac{u_0}{\sqrt{g \frac{\Delta \rho_0}{\rho} D}}$$

JET $F_{rd} \gg 1$

PLUME $F_{rd} \sim > 1$

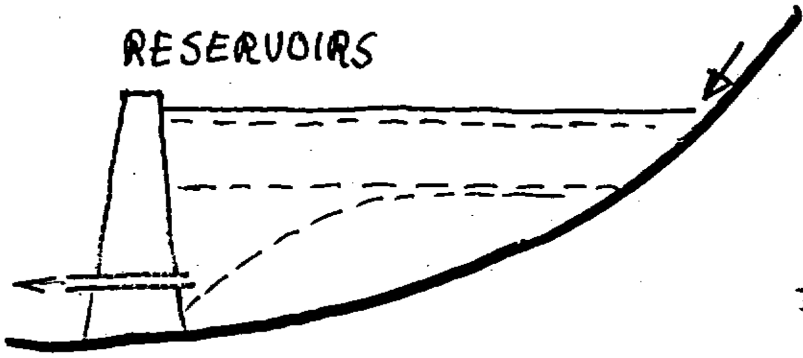
$F_{rd} < 1$



INTRUSION !

WITHDRAWAL FLOWS

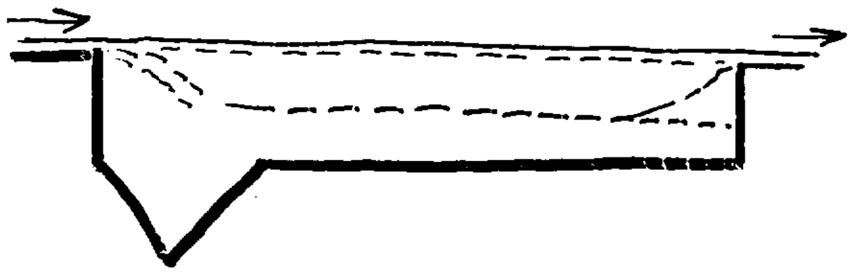
RESERVOIRS



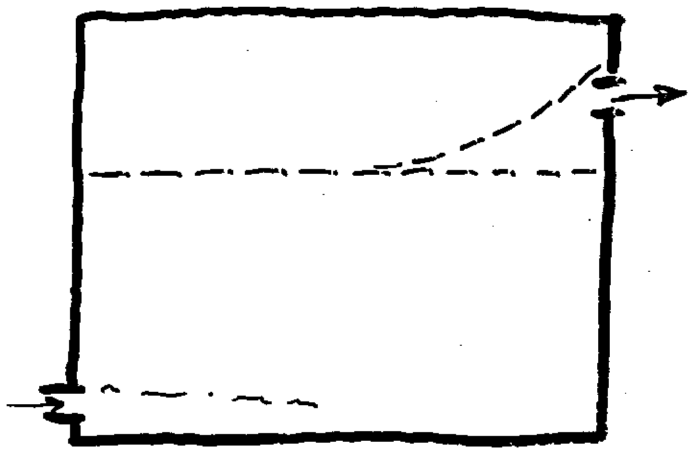
BERNOULLI EQ.

$$z\rho g + p + \frac{1}{2}\rho u^2 = \text{const}$$

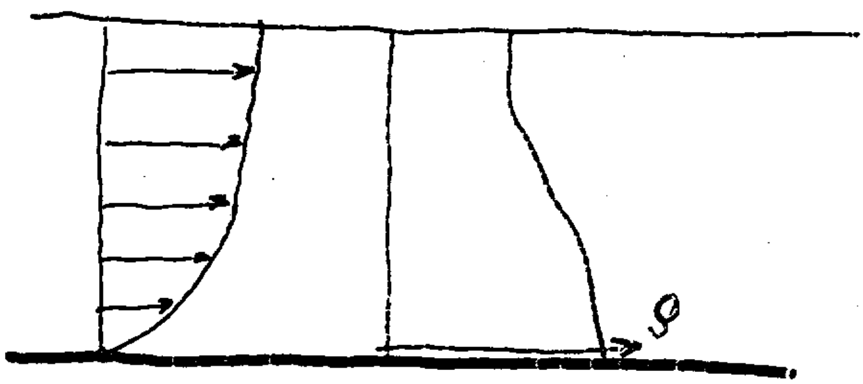
SETTLING TANKS



VENTILATED ROOMS ?



MIXING



$$Ri = \frac{-\frac{g}{\varrho}\frac{d\varrho}{dz}}{\left(\frac{du}{dz}\right)^2}$$

BUOYANCY FREQUENCY

$$\frac{D_z}{D_{z0}} = (1-\beta R_i)^{\alpha}$$

$\left.\begin{array}{l}\alpha = 1\\ \beta = 7\end{array}\right\}$ Stable cond.

Rodi [1984] $\left.\begin{array}{l}\alpha = -1/4\\ \beta = 14\end{array}\right\}$ Unstable cond.

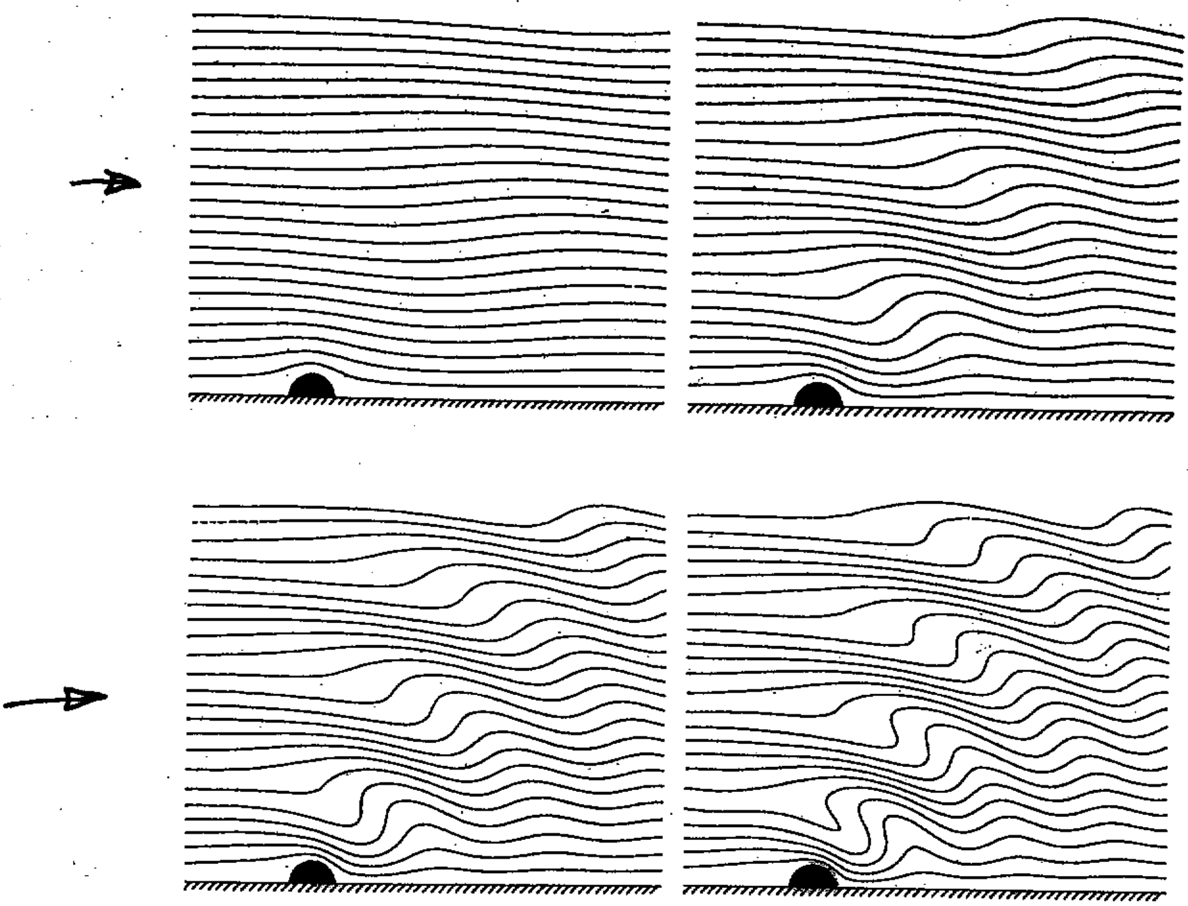


Figure 8. Lee waves excited in a half space by a semi-circular obstacle for κ = 0.5,1.0, κ_c (1.27) and 1.5. Taken from Huppert (1969), Figures A1-A4.

(a)

(b)

(c)

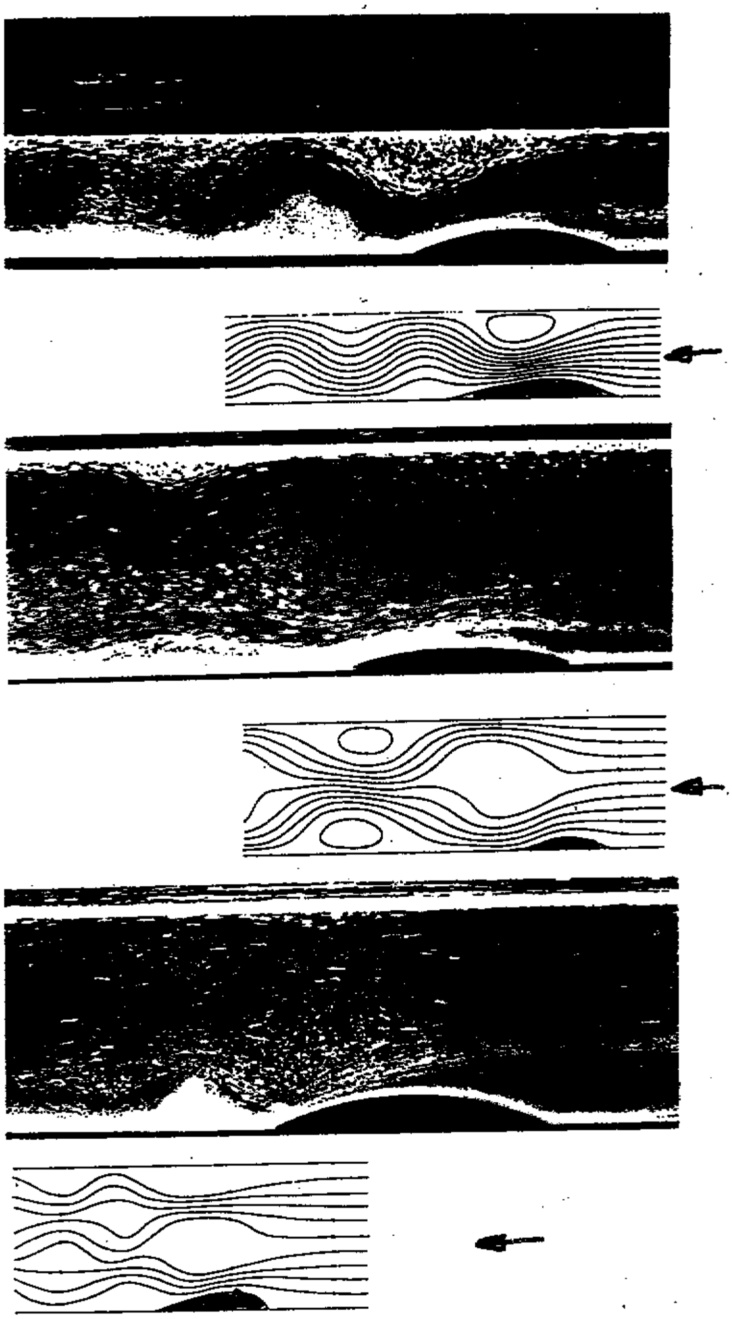
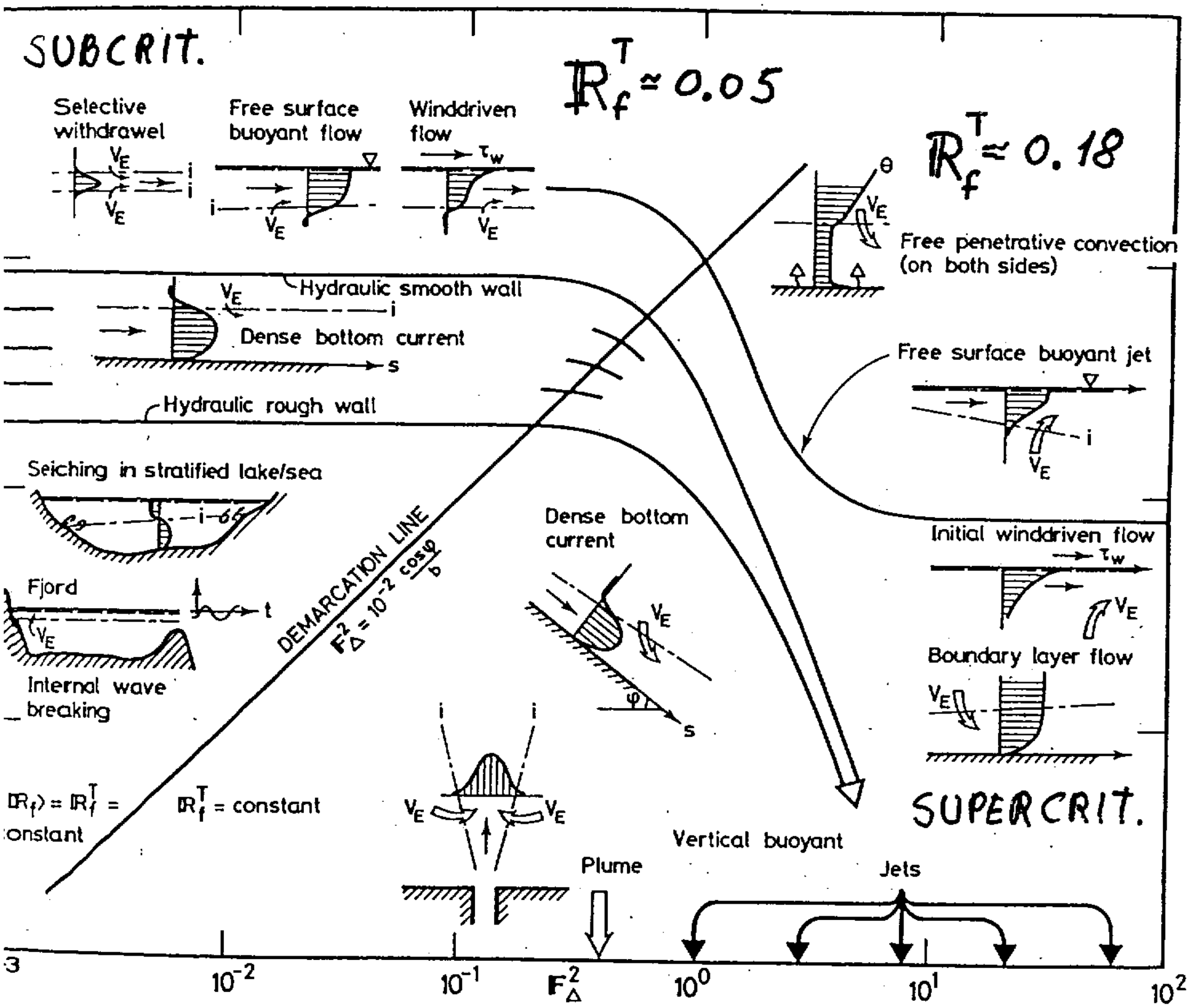


Figure 7. Lee waves excited by an obstacle in a channel. a) One mode, b) two modes, c) three modes. Taken from Long (1955), Figures 7-9.

[PEDERSEN, 1980]



$$\mathbb{R}_F^T = \frac{\text{kinetic energy used to mixing}}{\text{total generated kinetic energy}}$$

[PEDERSEN, 1980]

METHOD:

1. ENERGY BALANCE
2. BUCYANT PLUMES CORRECTED FOR MEANDERING

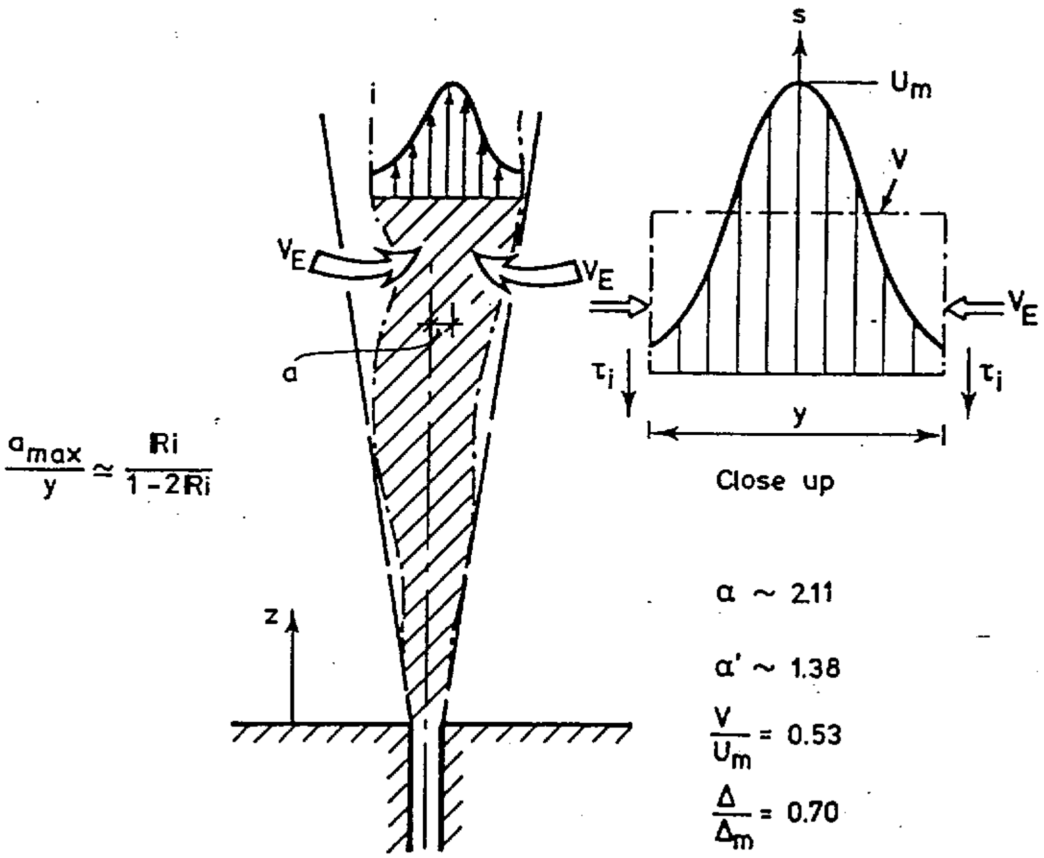


Figure 8.3. Definition sketch of a two-dimensional vertical buoyant jet. Shaded area: instantaneous position of the jet. Dashed lines: limits of the time averaged positions, i e of the meandering. The numbers are calculated on the basis of Bradbury's [1965] data, chapter 15.B.

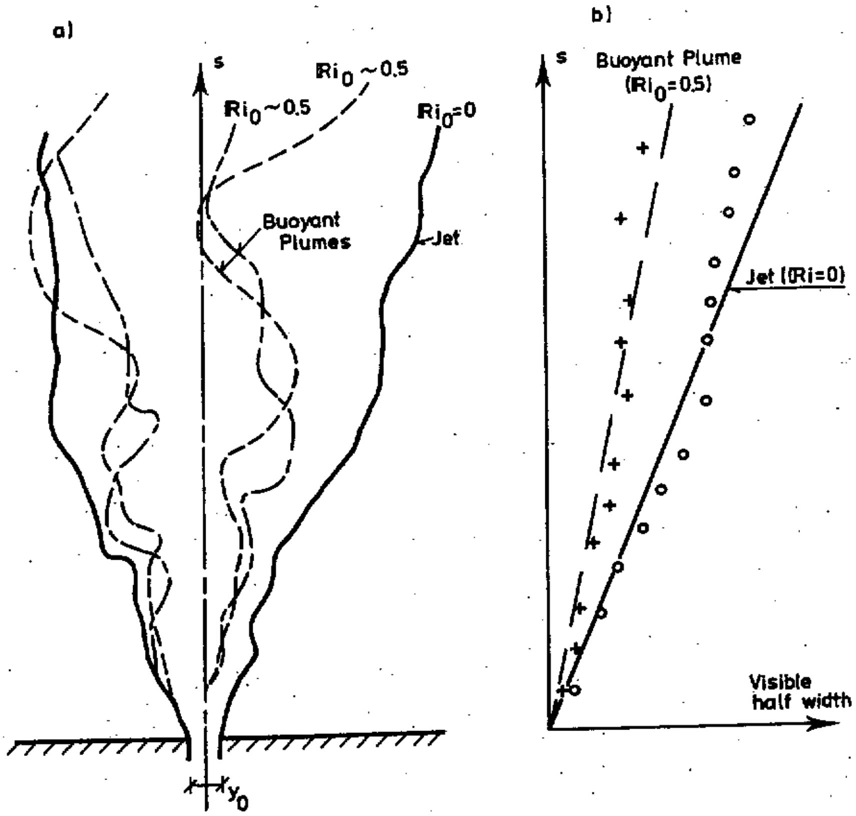


Figure 15.K1. a) Visible boundary of short-time (0.01 seconds) exposure photographs of a two-dimensional buoyant plume ($\mathrm{Ri} = 0.5$) and jet ($\mathrm{Ri} = 0$) respectively.
Photographs taken by Kotsovinos [1975].

(a)

(b)

(c)

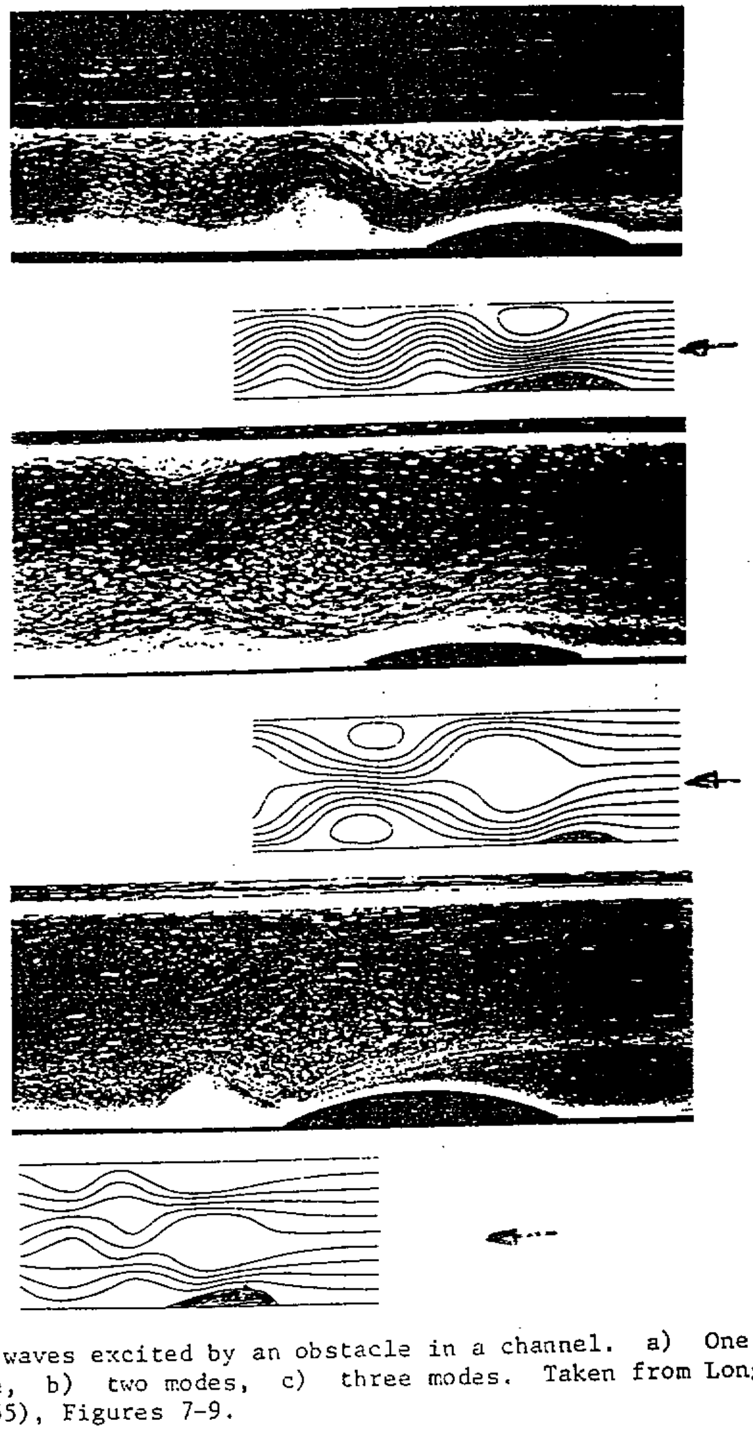


Figure 7. Lee waves excited by an obstacle in a channel. a) One mode, b) two modes, c) three modes. Taken from Long (1955), Figures 7-9.

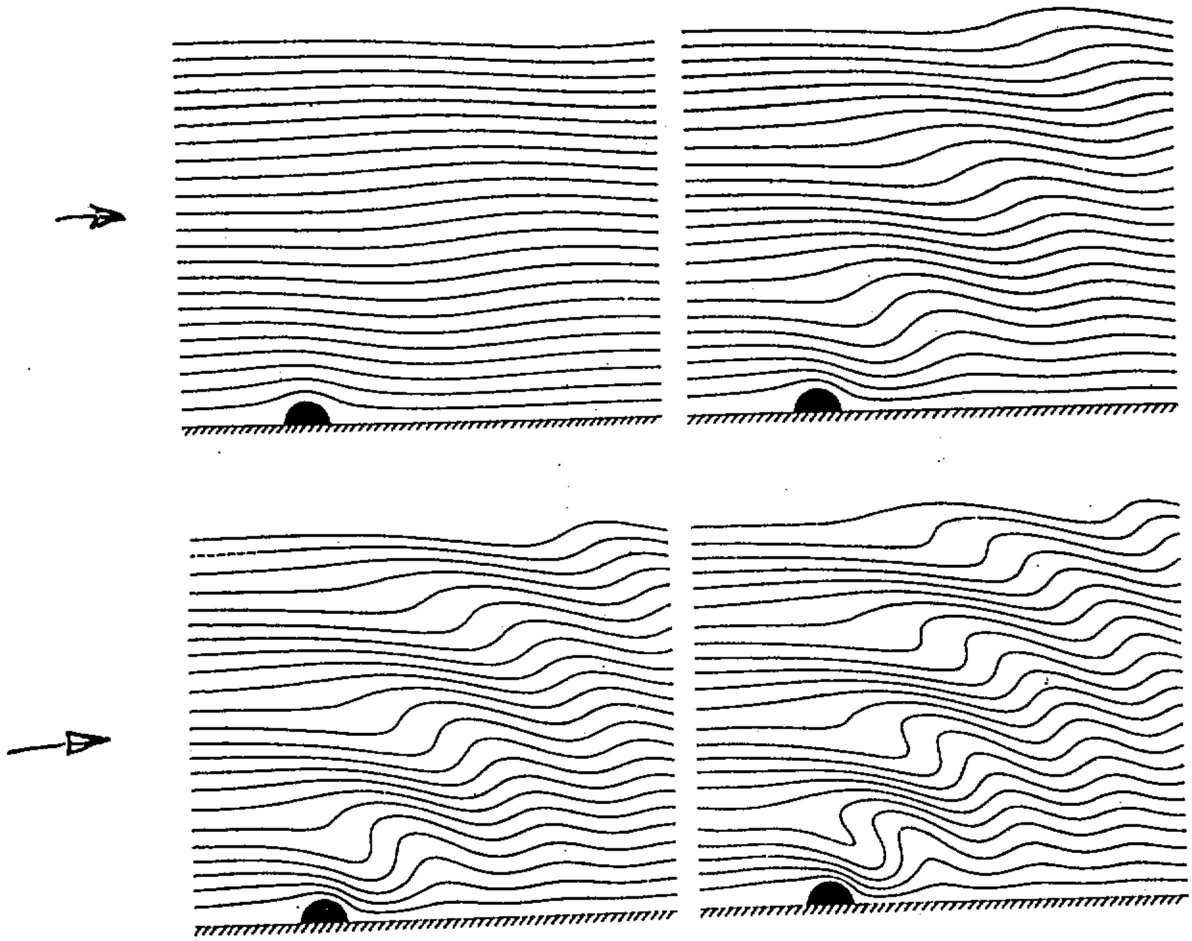
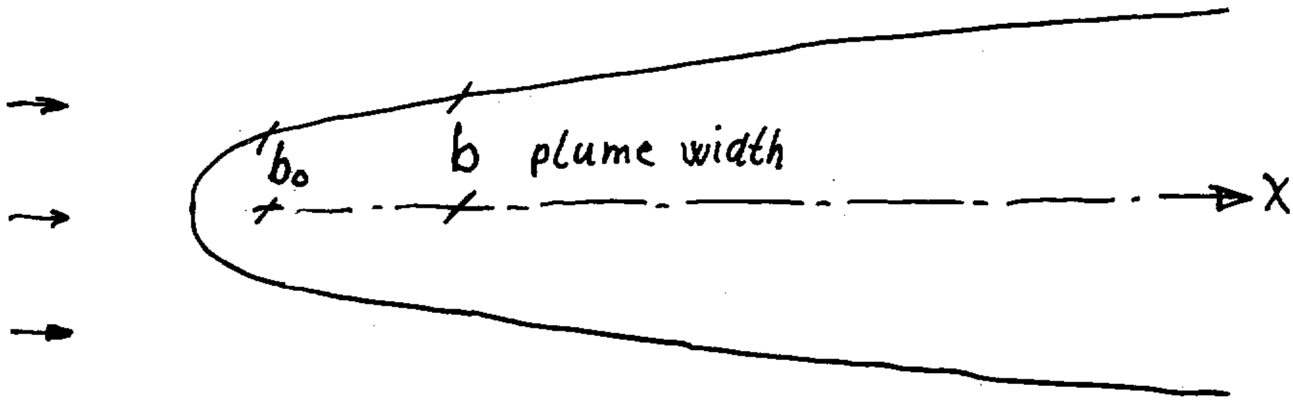


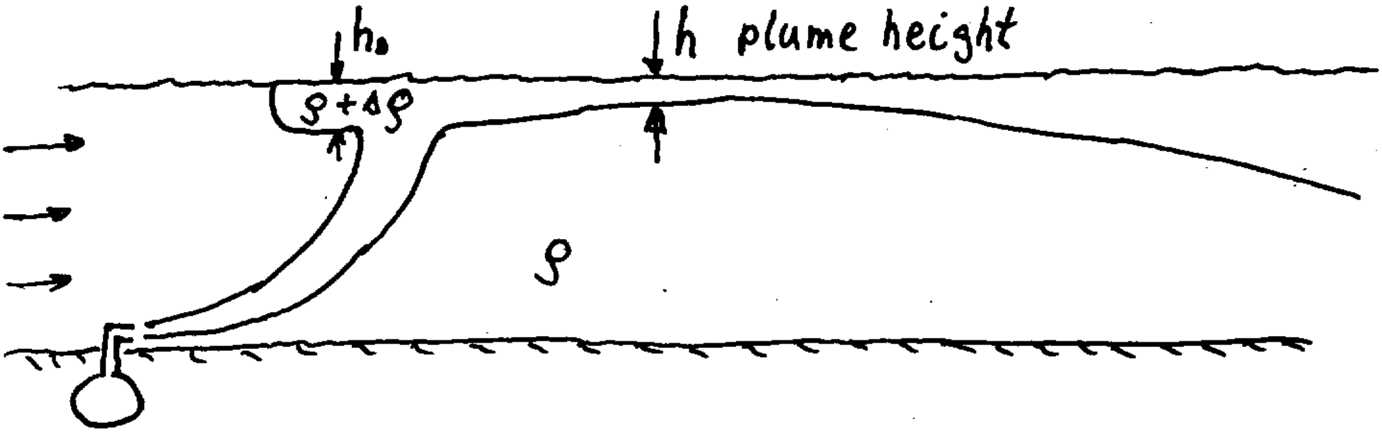
Figure 8. Lee waves excited in a half space by a semi-circular obstacle for κ = 0.5,1.0, κ_c (1.27) and 1.5. Taken from Huppert (1969), Figures A1-A4.

SURFACE PLUME MODEL

$$\frac{db}{dx} = \text{turbulent dispersion} + \text{buoyant spread}$$

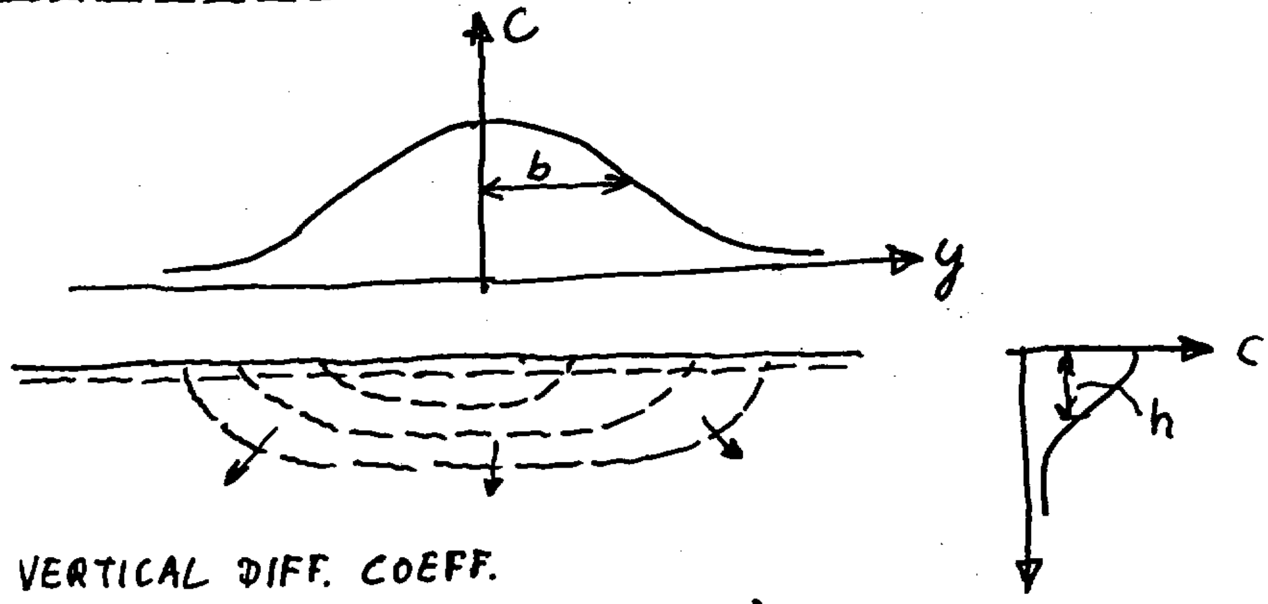


$$\frac{dh}{dx} = \text{turbulent dispersion} - \text{buoyant spread effect}$$



NUMERICAL SOLUTION E.G. BY A SPREAD SHEET PROGRAM (EG LOTUS 1-2-3)

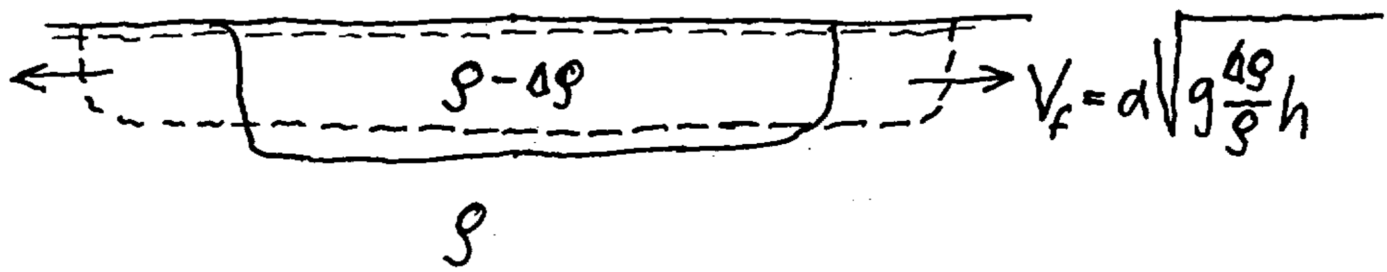
TURBULENT DIFFUSION



VERTICAL DIFF. COEFF.

$K_z = f$(ambient turbulence, $\Delta\rho$)

BUOYANT SPREAD



Turbulence Modeling in Stratified Flows Subject to Advective Buoyancy Fluxes

Jeffrey R. Koseff
Environmental Fluid Mechanics Laboratory
Department of Civil and Environmental Engineering
Stanford University
Stanford, CA 94305-4020
phone: (650) 36-2363 fax: (650) 725-9720 email: koseff@stanford.edu

Joel H. Ferziger
Department of Mechanical Engineering
Stanford University
Stanford, CA 94305-3030
phone: (650) 725-2080 fax: (650) 725-9720 email: ferziger@ecoule.stanford.edu

Stephen G. Monismith
Environmental Fluid Mechanics Laboratory
Department of Civil and Environmental Engineering
Stanford University
Stanford, CA 94305-4020
phone: (650) 723-4764 fax: (650) 725-9720 email: monismith@stanford.edu

Award #: N00014-03-1-0422
http://www.stanford.edu/group/efml/EFML.html

LONG-TERM GOALS

We are performing, in collaboration with Professor Mark Stacey of UC Berkeley, simulations of the mixing processes in stratified free-surface flows, which are typically found in the near-coastal domain. The goal of the proposed work is to determine which turbulence model will most efficiently and accurately predict the state of flow in estuaries and the coastal ocean, including both the current structure and the mixing coefficients. In particular, we are trying to determine whether turbulence closure parameterizations should be modified in the presence of sheared horizontal advection of a background horizontal density gradient.

OBJECTIVES

The primary question to be addressed is: How can we most accurately and most efficiently model turbulent mixing in estuaries and coastal circulation models where both vertical and horizontal density gradients are relevant. In particular, we are trying to determine whether turbulence closure parameterizations should be modified in the presence of sheared horizontal advection of a background horizontal density gradient. Other questions to be addressed include: How do shear, stratification, and turbulence interact in the presence of a longitudinal density gradient? What turbulence models perform best in the presence of a horizontal buoyancy flux? In a system with reversing advective buoyancy flux can models predict mixing and transport over long timescales?

APPROACH

In order to answer these questions, our joint approach (with Stacey at Berkeley) will involve a combination of Large Eddy Simulation (LES) and Reynolds-Averaged-Navier-Stokes (RANS) modeling. The large eddy simulation will allow us to examine the structure and dynamics of turbulence in our flow environment, and will produce data sets for use with the RANS modeling. The RANS model will be used to examine a variety of turbulence closures, and evaluate their performance against field data collected by Stacey and his group and the LES-produced data.

WORK COMPLETED

The project just got underway in March 2003. To date we have developed modifications for the constants in the k-ε turbulence model. In particular, we have developed parameterizations for the coefficients $c\mu$, Pr_t, $c\varepsilon_2$, and $c\varepsilon_3$ as a function of the turbulent Froude number and the turbulent Reynolds number, using the database developed in a previous ONR-sponsored project (see Shih et al., 2000, 2003). These parameterizations have been coded up and included in the GOTM code (Burchard et al., 1999) where their performance is being compared against the standard k-ε model and other versions of k-ε developed specifically for stratified flows.

RESULTS

The tests of the model were performed using the 1-D water column model GOTM, which is based on a modular format that enables it to be easily embedded in 3-D ocean circulation models and also allows for refinements or extensions for the turbulence models. We incorporated our proposed model into the GOTM code. Since none of the built in stability functions in GOTM modify their parameters based on depth, we had to adapt the code to modify the parameters in k-ε model discussed above to be depth varying based on the local turbulent Froude number.

The test case we use is a pressure gradient driven open channel flow in which the density is held fixed at both the lower solid boundary and the upper free surface. This allows the flow to develop to a fully developed steady state with a non-uniform density profile. The Reynolds number, Re_τ and the Richardson number, Ri_τ, (based on the shear velocity) were 682 and 31 respectively. Previous studies by Garg et al. (2000) have shown that LES have produced results that are in good agreement with DNS results for open channel flows. Hence, we use an LES run with identical conditions to those in our test case to assess the predictions of our proposed model and that of the standard k-ε model with constant stability functions. The flow was allowed to develop to a converged solution using the standard k-ε model after which the stratification was imposed. The velocity and density profiles at the initial state (i.e. after spin up; non-dimensional time tu_τ/h =0) are shown in Figure 1 for both the standard k-ε and the modified k-ε. Superimposed on these plots are the LES profiles.

The velocity and density profiles at a non-dimensional time of tu_τ/h =4.1 are shown in Figure 2. It is clear from this figure that the modified k-ε is doing a better job of capturing the density profile as well as the velocity profile. The velocity and density profiles at a non-dimensional time of tu_τ/h =14.6 are shown in Figure 3. It is clear from this figure that the modified k-ε is still doing a better job of capturing the density profile but that there seems to be an issue with both models in capturing the velocity profile. We are currently working on trying to resolve this issue.

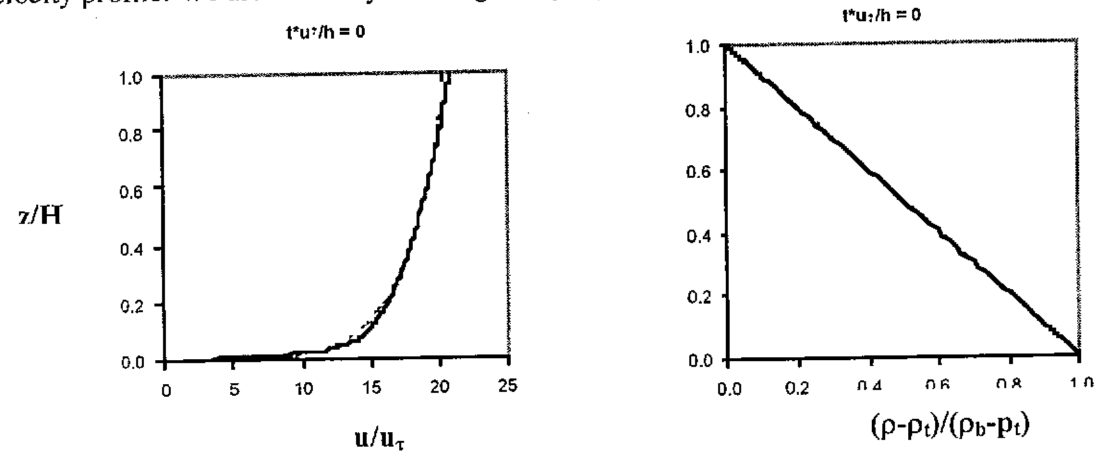


Figure 1: Plot of velocity and density profiles at tu_τ/h = 0 for the LES (solid line); standard k-ε (dashed line); and modified k-ε (dotted line)

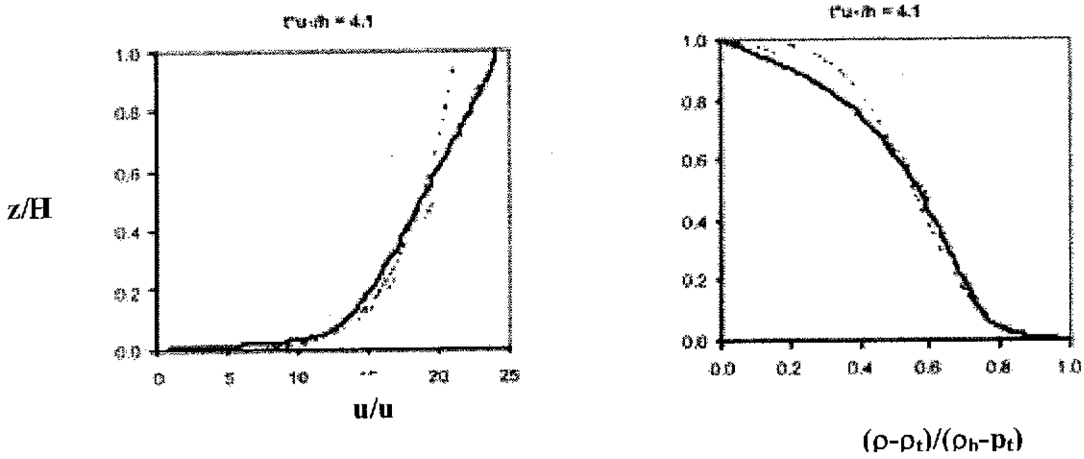


Figure 2: Plot of velocity and density profiles at $tu_\tau/h = 4.1$ for the LES (solid line); standard k-ε (dashed line); and modified k-ε (dotted line)

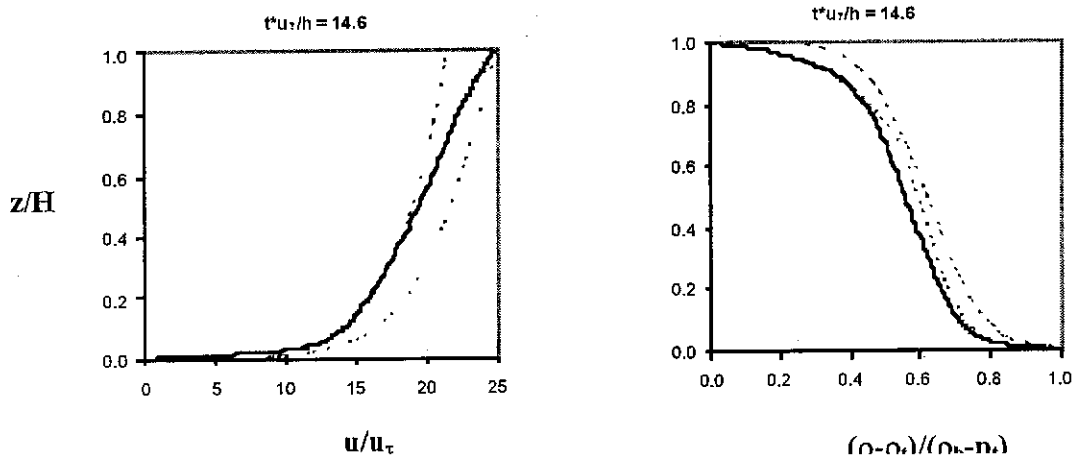


Figure 3: Plot of velocity and density profiles at $tu_\tau/h = 14.6$ for the LES (solid line); standard k-ε (dashed line); and modified k-ε (dotted line)

IMPACT/APPLICATIONS

The results demonstrate the intrinsic value of DNS and LES in that it allows us to calculate each term in a model or parameterization of the extant physics. Evaluation of existing turbulence closure models or commonly used sub-grid-scale parameterizations is therefore a lot more complete than with experiments alone.

TRANSITIONS

The numerical databases developed under separate ONR funding have been used in this project to develop new turubulence parameterizations for stratified flows, and to test these parameterizations. As part of our collaboration with Mark Stacey we are now discussing the development of an extensive field-measurement campaign (under separate funding) in South San Francisco Bay to develop an additional database to test the models developed during this project. This field campaign is motivated by the success to date of the approach we have adopted in this project. It will likely involve researchers from Stanford University, UC Berkeley, the USGS, and the University of Western Australia.

RELATED PROJECTS

Shear Production and Dissipation in a stratified tidal flow - ONR - (Monismith PI). Our fieldwork includes work on stratified tidal flows in which we are making Reynolds stress measurements using broadband ADCPs

An Experimental Study of a Breaking Interfacial Wave - NSF- (Koseff PI). In the laboratory we are performing experiments in an attempt to measure the mixing associated with a breaking internal wave at a stratified (two-layer) interface using the wave-generation technique of Rapp and Melville. In this work we are measuring the mixing efficiency associated with such an event.

REFERENCES

Burchard, H., Bolding, K., and Villareal, M.R. 1999 GOTM, a General Ocean Turbulence Model, Theory, Implementation, and Test cases, European Commission, Report EUR 18745, 103pp.

Garg, R. P., Ferziger, J. H., Monismith, S. G., and Koseff, J. R., 2000. "Stably Stratified Turbulent Channel Flows Part I: Stratification Regimes and Turbulence Suppression Mechanisms", *Physics of Fluids*, 12,10, pp. 2569-2594.

Shih, L.H., J.R., Koseff, J.H. Ferziger, and C.R. Rehmann, 2000. "Scaling and Parameterization of Stratified Homogeneous Turbulent Shear Flow." *Journal of Fluid Mechanics*, 412:1-20.

Shih, L.H., Koseff, J.R., Ivey, G.N., and J.H. Ferziger, 2003. "Prediction of fluxes from a homogeneous sheared stratified turbulence database", *Journal of Fluid Mechanics*, submitted.

PUBLICATIONS

Ferziger, J. H., Koseff, J.R., Shih, L.H., and S.K. Venayagamoorthy, 2003. "RANS models for stratified turbulence based on DNS data", *Physics of Fluids*, in revision.

Shih, L.H., Koseff, J.R., Ivey, G.N., and J.H. Ferziger, 2003. "Prediction of fluxes from a homogeneous sheared stratified turbulence database", *Journal of Fluid Mechanics*, submitted.

Venayagamoorthy, S.K., Koseff, J.R., Ferziger, J.H, and L.H. Shih, 2003. Testing of RANS turbulence models for stratified flows based on DNS data, Center for Turbulence Research, Stanford University, Annual Research Briefs, submitted for publication.

Next Up Previous
Next: Introduction

Mixing and Available Potential Energy in Stratified Flows

Yu-heng Tseng and Joel H. Ferziger
Environmental Fluid Mechanics Laboratory, Stanford University
Stanford, CA 94305-4020, USA
Corresponding author: 650-725-2085 (tel), 650-725-3525 (fax), yhtseng@stanford.edu

Abstract:

Mixing plays an important role in atmospheric and oceanic flows. Mixing occurs on small scales and is due to molecular diffusion. It is irreversible while stirring is a kinematic process that enhances mixing but is reversible. Energy budgets used to investigate these processes include that of the available potential energy, for which the reference potential energy, a measure of diapycnal mixing, must be computed. We develop an approach for calculating the available potential energy from the probability density function that is more efficient than existing methods, especially in two and three dimensions and is suitable for both numerical simulation and experiments. A new length scale is defined to quantify stirring and provides a useful measure of overturns resulting from stirring. A simulation of lid-driven cavity flow provides an illustration of the method. It will later be extended to other flows.

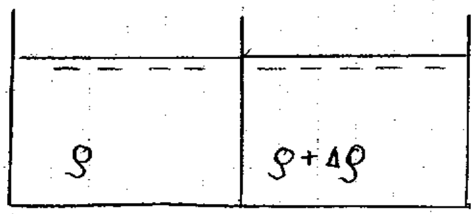
12.8.2004 / TL

a note on

LOCK-EXCHANGE EXPERIMENT

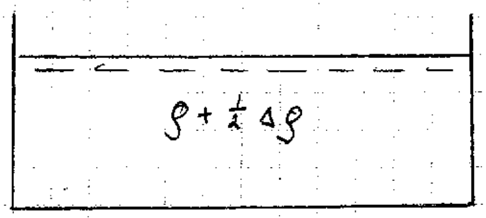
MIXING EFFICIENCY CHARATERIZED BY THE POTENTIAL ENERGY OF THE STRATICATION

Initial condition



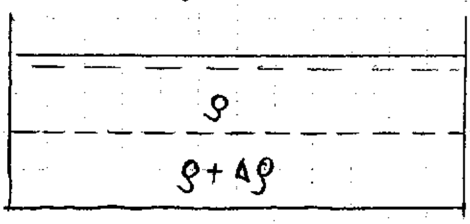
$$E_{pot} = \left[\frac{h}{2}\rho + \frac{h}{2}(\rho+\Delta\rho) \right] g$$
$$= \left[h\cdot\rho \quad \frac{h}{2}\Delta\rho \right] g$$

Fully mixed



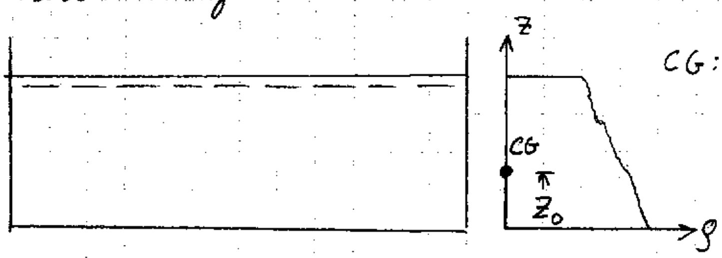
$$E^F_{pot} = \left[2\,\frac{h}{2}(\rho + \tfrac{1}{2}\Delta\rho) \right] g$$
$$= \left[h\rho + \frac{h}{2}\Delta\rho \right] g$$

No mixing



$$E^N_{pot} = \left[\frac{h}{4}(\rho+\Delta\rho) + \frac{3}{4}h\rho \right] g$$
$$= \left[h\rho + \frac{h}{4}\Delta\rho \right] g$$

Real mixing



CG: centre of gravity

$$E^R_{pot} = \left[\rho + (\rho+\Delta\rho) \right] \cdot z_0\, g$$

$$\text{Degree of mixing} = \frac{E^F - E^R}{E^F - E^N}$$